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Constraints on quantum hidden-variables and the Bohm theory

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Abstract. In this paper the no-hidden-variables arguments of Mermin and Peres are assessed in the light of the Bohm formulation of quantum theory.

1. Introduction

In order to resolve the question as to whether quantum mechanics can be considered complete, various authors have provided arguments which attempt to constrain possible hidden-variable theories. These arguments make assumptions that delimit the possible character of the hidden variables and then go on to demonstrate that such restricted hidden variables are incapable of providing the desired completion of the quantum theory. Now it is well known that a perfectly consistent hidden-variable formulation of quantum mechanics in fact exists: the de Broglie–Bohm theory [1, 2]†. And just as the no-hidden-variable arguments tell us the features any theory underlying quantum theory may not possess, the de Broglie–Bohm approach gives an insight into the features that a more complete quantum theory may indeed possess. Furthermore, if one looks at the proposed no-hidden-variable arguments from the Bohm point of view the restrictive assumptions that these arguments entail can be easily identified.

In this paper the no-hidden-variables arguments of Mermin [3] and of Peres [4] will be considered. These arguments concern spin measurements carried out on two spin-$\frac{1}{2}$ particles. The aim of both arguments is to show, for a given set of operators, not all of which commute, that it is not possible for certain types of hidden-variable theory to assign values (or measured outcomes) for all of the operators without inducing an algebraic contradiction. Both authors are aware that their arguments do not rule out all types of hidden-variable theories and it is hoped that this paper will complement their work by examing the reasons why the arguments they propose do not apply to the de Broglie–Bohm interpretation. To facilitate our discussion we begin with a description of spin and its measurement according to the Bohm theory.

† In fact there are significant differences between the theory of Bohm and that of de Broglie, especially concerning the many-body case.

2. The measurement of a spin component in the Bohm theory

An essential feature of the Bohm theory is the way it handles measurement and to facilitate our discussion the Bohm, Schiller and Tiomno (BST) [5] approach to the spin will be discussed in the context of the measurement of a component of the spin of a single particle. Full details may be found in [6]. In the Bohm approach a particle has a well-defined position $\boldsymbol{x}$ at all times. The particle trajectory is determined, once its (uncontrollable) initial position and its initial wavefunction are given, through the integral curves of the guidance formula $\boldsymbol{v} = \boldsymbol{j}/\rho$, where $\boldsymbol{j}$ is the usual current and ρ the probability density. All observable quantities associated with the particle have well-defined values determined by the actual position of the particle and the wavefunction. It is important to understand that only those observables for which the state is an eigenstate have values that are eigenvalues. In general the value of an observable can take a continuum of (hidden) values, the act of measurement or preparation transforms the existing value into an eigenvalue. A discussion of this point in the context of quantum transitions is given in [7]. This feature can also be seen clearly in the following analysis of spin measurement

In the BST theory the Pauli spinor is interpreted as defining a state of rotation given by the Eulerian angles θ, ϕ and χ through writing

$$\psi = R\left\{\cos(\tfrac{1}{2}\theta)\exp\left[\tfrac{1}{2}\mathrm{i}(\phi+\chi)\right]u_+ + \mathrm{i}\sin(\tfrac{1}{2}\theta)\exp\left[-\tfrac{1}{2}\mathrm{i}(\phi-\chi)\right]u_-\right\} \tag{1}$$

where R is a complex spatial amplitude and $\sigma_z u_\pm = \pm u_\pm$ The probability density is

$$\rho = \psi^\dagger\psi \tag{2}$$

and the current

$$\boldsymbol{j} = \frac{\hbar}{2mi}(\psi^\dagger\nabla\psi - (\nabla\psi^\dagger)\psi) - \frac{e}{mc}\boldsymbol{A}\rho \tag{3}$$

yields the velocity

$$\boldsymbol{v} = \frac{\hbar}{mc}(\nabla\chi + \cos\theta\nabla\phi) - \frac{e}{mc}\boldsymbol{A}. \tag{4}$$

The value of the spin is given by

$$\boldsymbol{s} = \tfrac{1}{2}\hbar(\psi^\dagger\sigma\psi/\rho) = \tfrac{1}{2}\hbar(\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta). \tag{5}$$

To discuss spin measurement the time-dependent Pauli equation can be solved for the Stern–Gerlach (SG) magnetic field given some initial state. The particle trajectories can then be calculated by integrating equation (4) whilst the values of the spin components can be calculated from (5).

Consider the initial state

$$\psi = f_0(z)(c_+u_+ + c_-u_-) \tag{6}$$

where f_0 is a Gaussian packet centred at $z = 0$ and we ignore the motion in the x and y directions. c_+ and c_- are arbitrary constants. In this state the spin is independent of position. The values assigned to the spin components are

$$\begin{aligned} s_z &= c_+^2 - c_-^2 \\ s_x &= c_+^\star c_- + c_-^\star c_+ \\ s_y &= \mathrm{i}(c_-^\star c_+ - c_+^\star c_-). \end{aligned} \tag{7}$$

Clearly these need not be the eigenvalues of the operators. If $c_+ = c_- = 1/\sqrt{2}$ and the measurement is carried out in the z direction then during the process the state of the system is transformed to

$$\psi = f_+(z)u_+ + f_-(z)u_- \tag{8}$$

where f_+ and f_- are two packets which separate in the z direction. The set of possible trajectories for this situation bifurcate along the plane $z = 0$. Those trajectories with initial position $z < 0$ enter the lower packet where $\sigma_z = -1, \sigma_x = 0$ and $\sigma_y = 0$, whilst those with $z > 0$ enter the upper packet where $\sigma_z = -1$, $\sigma_x = 0$ and $\sigma_y = 0$. During the measurement the spin and position become coupled and as the particle follows its trajectory the spin evolves continuously from $\sigma_x = 1$, $\sigma_y = 0$ and $\sigma_z = 0$ to $\sigma_z = \pm 1, \sigma_x = 0$ and $\sigma_y = 0$†. If one now makes a measurement of σ_x on the f_+ packet this packet splits along the plane $x = 0$, into two separating packets. One packet is associated with $\sigma_z = 0$, $\sigma_x = 1$ and $\sigma_y = 0$ and the other with $\sigma_z = 0$, $\sigma_x = -1$ and $\sigma_y = 0$.

The manner in which a measurement disturbs the system is clear and calculable in the Bohm approach. Evidently there can be no ensembles dispersion-free in each of σ_z, σ_x and σ_y simultaneously. The unavoidable distribution of particle positions within the wavepacket ensures that this is so. It is also clear that measurements do not simply reveal pre-existing values in the Bohm theory and so there are no faithful measurements in quantum theory.

3. Spin measurements on two spin-$\frac{1}{2}$ particle systems

Two (possibly widely separated) SGs are arranged such that SG2, which measures a spin component of particle two, is fixed in orientation along a z axis whilst SG1, which measures a spin component of particle one, may be rotated around the y axis by the angle δ. The direction in which SG1 is oriented shall be called z'. The coordinates z_2 and z'_1 are those that are significant from the point of view of inferring the value of the spin component in those measured directions. If after the measurement either z_2 or z'_1 turn out to be positive (negative) the value $+1(-1)$ (in units of $\hbar/2$) is assigned to the z or z' component of the spin of that particle. (One might say that these components form the apparatus coordinates since by observing them the value of the spin may be inferred). Using only the z' direction for particle one and the z direction for particle two, one can construct a section through the configuration space of the two-particle system that is two dimensional and thus visualizable. In this discussion the evolution of the probability density will be displayed in this section of the configuration space. The two-particle Pauli equation can be solved with the two SG fields for the time development of the Pauli spinor from a given initial state. The details of this calculation for the initial state (13) are given in [8] here we present the results in a somewhat novel way which brings out the interesting features more clearly.

Firstly let us assume that the wavefunction of the two particle system at the entrance to the magnets is given by a product of a product spatial wavefunction and the singlet spin state.

$$\Psi_0 = f_1(z'_1)f_2(z_2)\frac{1}{\sqrt{2}}(u_+v_- - u_-v_+) \tag{9}$$

† Space prohibits the use of figures, which are given in [6].

where f_1 and f_2 are normalized gaussian packets, z_1' and z_2 are the coordinates of particles 1 and 2 in the z' and z directions respectively and $\sigma_{z_1} u_\pm = \pm u_\pm$, $\sigma_{z_2} v_\pm = \pm v_\pm$. This is the Einstein–Podolsky–Rosen–Bohm state.

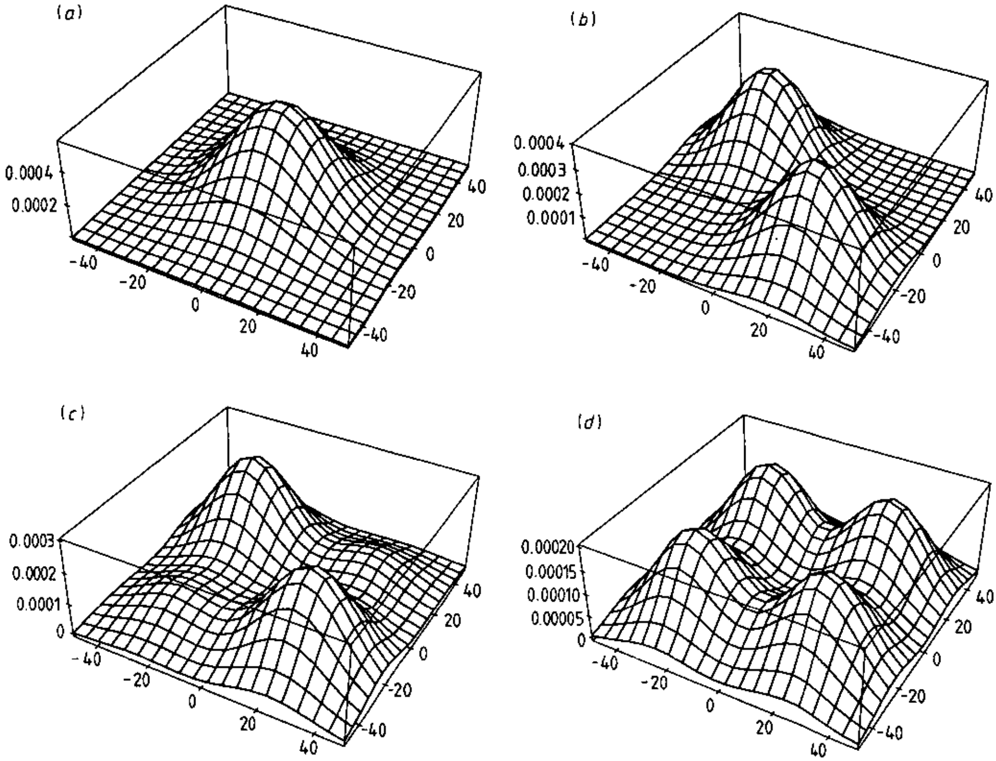

Figure 1. The evolution of the probability density in the configuration space section spanned by z', z. (*a*) Initial probability density. (*b*) Final probability density $\delta = 0$. (*c*) Final probability density $\delta = \pi/3$. (*d*) Final probability density $\delta = \pi/2$.

Figure 1(*a*) shows the initial probability density in the configuration-space section defined above for the state (9). Figures 1(*b*)–(*d*) show the probability density after the measurements are carried out when the two SGs are aligned ($\delta = 0$), when rotated by $\pi/3$ and when rotated by $\pi/2$, respectively. From these figures it is clear that altering the angle of one of the magnets, when the system is in the singlet state, alters the evolution of the system as a whole. Viewing the system in this way gives us a pictorial representation of the quantum feature of the indivizibility of many particle quantum systems so strongly emphasized by Bohr. This indivisibility would normally imply connection between the particles but in the usual formulation of quantum theory there is no way to discuss the evolution of the two particles separately.

The use of configuration space in quantum mechanics is not a matter of convenience, as it is in classical mechanics. In general it is not possible to define separate wavefunctions, one for each particle, each of which obeys a separate and local single particle equation, which taken together can reproduce the behaviour of the system. The exception to this, of course, is when the configuration space wavefunction factorizes, in this case an equivalent pair of individual equations can be written down, individual wavefunctions can be defined unambiguously and the two particles behave independently. Figure 1(*d*) can be interpreted to apply in this case. If the initial spin state of the two particle system is a product of the eigenstates corresponding to $\sigma_x = 1$ for both particles and if the two measurements are carried out in the z direction on each particle then the probability density develops as shown in figure 1(*d*).

4. Spin measurements on two spin-$\frac{1}{2}$ particle systems, the Bohm theory

For the two-particle case one takes the natural extensions to the definitions given for one particle

$$v_i = \frac{-\mathrm{i}\hbar}{2m\rho}\psi^\dagger \overleftrightarrow{\nabla}_i \psi - \frac{e}{mc}A \qquad i = 1,2 \tag{10}$$

$$s_i = \frac{\hbar}{2}\psi^\dagger \sigma_i \psi/\rho \qquad i = 1,2 \tag{11}$$

$$s_i s_j = \frac{\hbar^2}{4}\psi^\dagger \sigma_i \sigma_j \psi/\rho \qquad i = 1,2 \quad j = 1,2. \tag{12}$$

From these equations it is easily seen that given x and the quantum state ψ all the spin component operators and all of their products have well-defined values, but not all of these values may be eigenvalues simultaneously. It is this fact that enables the Bohm theory to avoid the algebraic contradiction which arises when one attempts to assign values to individual operators and their products, such as those given by Mermin and Peres.

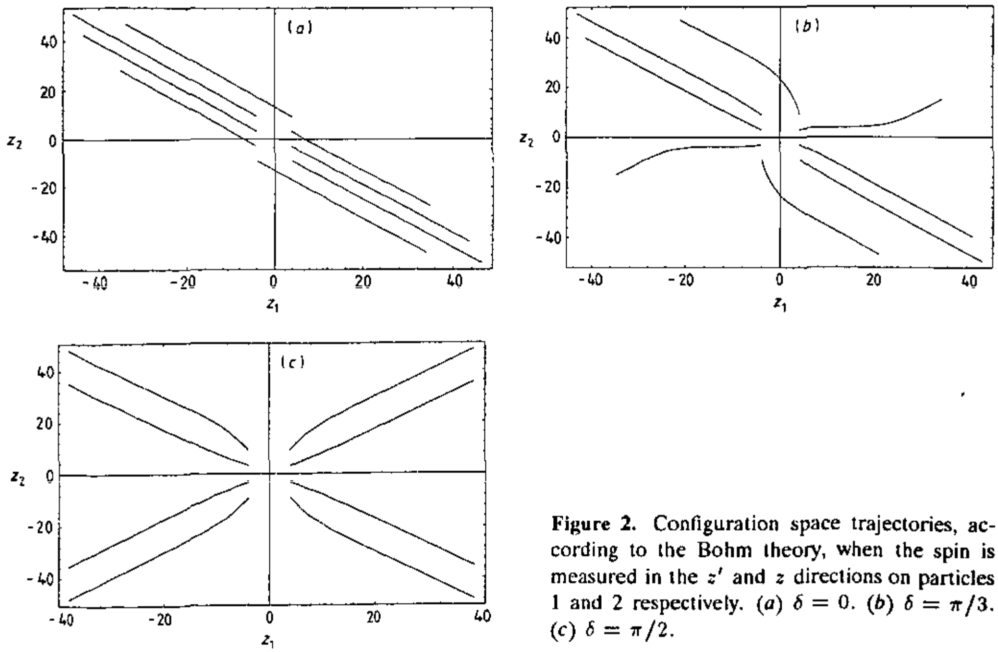

Figure 2. Configuration space trajectories, according to the Bohm theory, when the spin is measured in the z' and z directions on particles 1 and 2 respectively. (*a*) $\delta = 0$. (*b*) $\delta = \pi/3$. (*c*) $\delta = \pi/2$.

The results of the application of the Bohm theory to this case are discussed here in terms of the reduced configuration space trajectories associated with figures 1(*a*)–(*d*). For clarity of presentation a set of initial positions of the two particles is chosen with $z_1' = +4$ or -4, whilst $z_2 = -9, -3, 3, 9$, (in arbitrary units) this gives eight trajectories in all. Figures 2(*a*)–(*c*) show the configuration space trajectories associated with figures 1(*b*)–(*d*) respectively. Figure 2(*a*) shows the trajectories for the case in which $\delta = 0$. The outcome of the measurement in the fixed z direction on particle two, (whether z_2 is positive or negative), for a given initial value of z_2, can depend on the initial value of z_1' in addition. For example the trajectory that starts at $z_2 = -4, z_1' = +4$, ends with z_2 negative (and hence spin -1), whilst that trajectory that starts with $z_2 = -4, z_1' = -4$ ends with z positive (and hence spin $+1$). A similar dependence of the outcome for particle one on the initial position of particle two is also evident in figure 2(*b*).

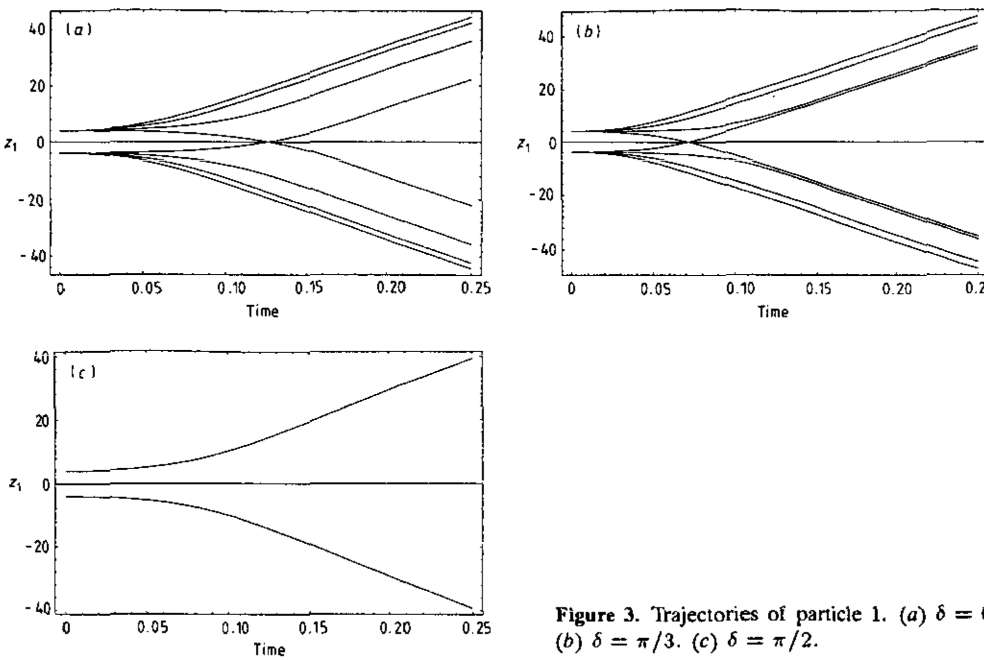

Figure 3. Trajectories of particle 1. (*a*) $\delta = 0$. (*b*) $\delta = \pi/3$. (*c*) $\delta = \pi/2$.

Figure 2(*b*) differs from figure 2(*a*) only in that the angle of SG1 is altered, $\delta = \pi/3$. Figure 2(*c*) shows that when $\delta = \pi/2$ the individual particle trajectories are independent. This is to be expected since there are no correlations in this case. In figure 2(*c*) any trajectory which starts from $z_2 < 0$, ends with $z_2 < 0$ (spin -1), regardless of the position of particle one. When $\delta = 0$ (figure 2(*a*)), the trajectory starting at $z_2 = -4, z_1 = -4$ ends with z_2 positive (spin $+1$). It is clear from the differences between these figures that altering the orientation of SG1 can result in a different outcome for particle two even when the initial positions of both particles are held constant. The individual particle trajectories with the evolution of the coordinates with time are plotted in figures 3 and 4 respectively and are labelled (*a*), (*b*) and (*c*), corresponding to $\delta = 0, \pi/3$ and $\pi/2$, respectively.

As previously pointed out, if the initial spin state is a product of the eigenstates of σ_x with eigenvalue 1 for each particle and the spin component in the z direction

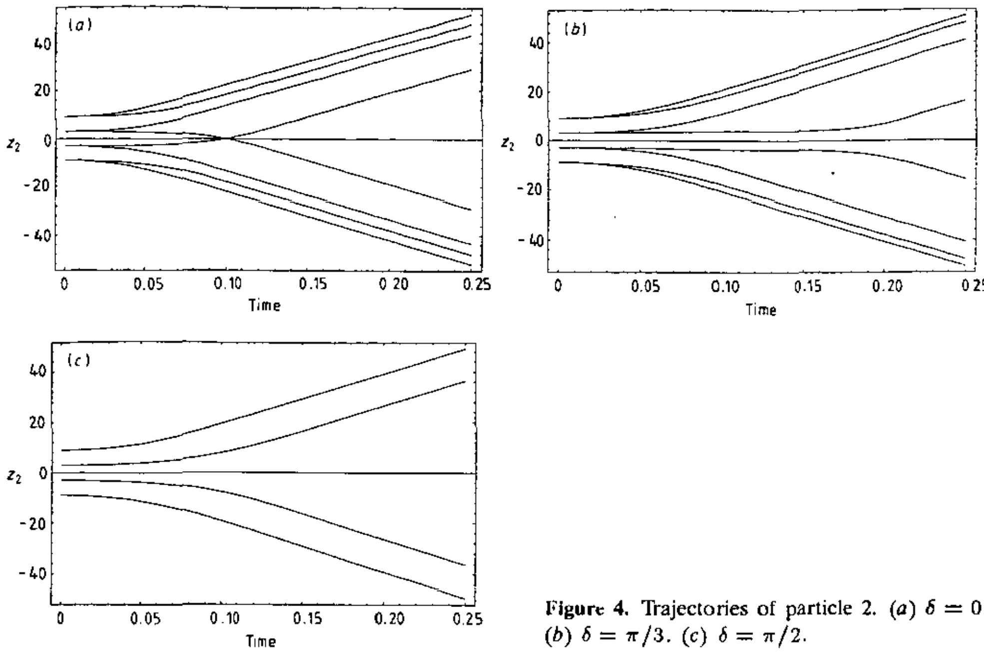

Figure 4. Trajectories of particle 2. (*a*) $\delta = 0$. (*b*) $\delta = \pi/3$. (*c*) $\delta = \pi/2$.

on each particle is measured figure 2(*c*) applies. For a given initial position for particle two a unique outcome is defined which is independent of the initial position of particle one and the angle of SG1.

Consider now the set of operators $\sigma_{1x}, \sigma_{2x}, \sigma_{1x}\sigma_{2x}$ and the product of these, the identity I. Clearly the eigenvalue of I is 1. The values assigned to the σ operators in the Bohm theory according to (11) and (12) are zero in the state (9). Although the operators obey a product rule the assigned values in general do not. Of course, if the state of the system is an eigenstate of the set of operators the product rule will also hold for the assigned values.

To summarize, in the Bohm theory we see that for spin measurements on two spin-$\frac{1}{2}$ particles:

1. the principle of faithful measurement does not apply, values assigned to observables are only eigenvalues when the state of the sysytem is an eigenstate of the corresponding operator;
2. there are no ensembles dispersion free for each of a set of non-commuting operators;
3. relations holding amongst operators need not hold for the values assigned them except when the state is a simultaneous eigenstate of the operators concerned;
4. when the state of the system is an entangled state the outcome of a measurement on a single particle can depend on the position of the other particle;
5. when the state of the system is an entangled state the outcome of a measurement on a single particle, for a given set of positions, can depend on which (if any) measurements is carried out on the other particle; and
6. when the state of the system is a simple product the outcome of a measurement on one of the particles is independent of both the position of the other particle and the particular choice of measurements that may be made on the other particle.

With these points in mind let us now examine the arguments given by Mermin and by Peres.

5. The arguments of Mermin and of Peres

Both arguments concern sets of operators defined on a system of two spin-$\frac{1}{2}$ particles. Mermin considers the following set of nine operators

$$\begin{array}{ccc|c} \sigma_{1x} & \sigma_{2x} & \sigma_{1x}\sigma_{1x} & 1 \\ \sigma_{2y} & \sigma_{1y} & \sigma_{1y}\sigma_{2y} & 1 \\ \sigma_{1x}\sigma_{2y} & \sigma_{1y}\sigma_{2x} & \sigma_{1z}\sigma_{2z} & 1 \\ \hline 1 & 1 & -1 & \end{array}$$

The operators in each row and in each column commute. The product of the values of the operators in each row is 1, as indicated. The product of the values of the operators down each column is also 1 except in the last column for which the product is -1. The eigenvalue of each of the operators must be $+1$ or -1, but it is clearly not possible to assign one of the values, $+1$ or -1, to each operator in a manner consistent with the products shown. In straight quantum mechanics this is of course not surprising. There is no state which is a simultaneous eigenstate of the operators (they do not all commute) and hence not all of them can have well-defined values simultaneously. Any set of operators in a row or a column (except the last) can have simultaneously well-defined values (if the state of the system is a simultaneous eigenstate) but only at the expense of making the values of the other non-commuting operators undefined. So much is elementary quantum mechanics.

In the many-worlds approach to quantum mechanics one might say that there can be no contradiction since each row or column of operators can only have well-defined values in a different universe.

We need to distinguish two possible interpretations of Mermin's argument. One, a strong interpretation, in which the values assigned to the operators in the table, by the hidden-variable theory, are taken to be the values that a given system actually possess simultaneously (whether measured or not) in a given state. The other, a weaker interpretation, in which the values assigned to the operators in the table are the values that would be obtained when the system is in some state with specific hidden variables and a measurement is carried out. These values could be called counterfactual and (since there is no simultaneous eigenstate of all the operators) they may not be attributed jointly at any time to an individual system.

To produce the desired contradiction a number of assumptions are necessary concerning the values assigned by the hidden-variable theory.

1. The values assigned to the operators in the table must be eigenvalues.
2. Relationships obeyed by operators must always be obeyed by the assigned values.
3. There is a unique value for each operator that does not depend on which other operators (if any) are measured on the system simultaneously.
    (a) The value assigned to an operator for one particle does not depend on which measurement is carried out on the other particle.
    (b) The values assigned to the product operators do not depend on which other operators (if any) are measured on the system simultaneously.

The strong form of the argument requires assumptions 1, 2 and 3. It fails for any hidden-variable theory in which assumption 1 does not hold, as is the case in the Bohm theory. If 1 does not hold then 2 need not.

In the weak form of the argument 1 and 2 hold trivially. The third assumption need not hold in a hidden-variable theory. Indeed it does not in the Bohm theory.

Now Mermin's argument does not depend on the state of the system, but the system will be in some state. If one assumes this state to be the product state

$$\psi = f_a(z_1)f_b(z_2)u_+(1)u_-(2) \tag{13}$$

or any similar state, then the Pauli equation splits into two independent equations and there can be no correlations between measurements carried out on particle one and measurements carried out on particle two separately. Assumption 3a holds trivially for (13). Let us examine Mermin's argument from the Bohm point of view. If we assume that the system is in the quantum state (13) then the values assigned to the operators in the table (in the strong sense) using equations (11) and (12) are

$$\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \hline 1 & 1 & -1 & \end{array}$$

The state (13) is not a simultaneous eigenstate of any row or column of operators and so although the product relationship amongst the operators are true these relationships do not hold for the actual assigned values. Indeed the values are not eigenvalues.

Now let us assume that starting in state (13) with hidden variables $x_1, y_1, z_1 > 0$ and $x_2, y_2, z_2 > 0$ a measurement of $\sigma_{1x}$and σ_{2y} is carried out. If we assign a value to $\sigma_{1x}\sigma_{2y}$ that is simply the product of the individual values the table has the following form

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 1 & -1 & \end{array}$$

If $\sigma_{1x}\sigma_{2y}, \sigma_{1y}\sigma_{2x}$ and $\sigma_{1z}\sigma_{2z}$ are measured simultaneously, after the interaction the state of the system will be a simultaneous eigenstate of these operators. Given the initial state (13) the final state will be either

$$\frac{1}{\sqrt{2}}(u_+(1)u_+(2) + \mathrm{i}u_-(1)u_-(2)) \tag{14}$$

with eigenvalues $1, 1, 1$ for $\sigma_{1x}\sigma_{2y}, \sigma_{1y}\sigma_{2x}$ and $\sigma_{1z}\sigma_{2z}$ respectively, or

$$\frac{1}{\sqrt{2}}(u_+(1)u_+(2) - \mathrm{i}u_-(1)u_-(2)) \tag{15}$$

with eigenvalues $-1, -1, 1$ for for $\sigma_{1x}\sigma_{2y}, \sigma_{1y}\sigma_{2x}$ and $\sigma_{1z}\sigma_{2z}$ respectively.

The specific outcome in an individual set of measurements will be determined by the hidden variables, but the table in a specific case could have the following form

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & -1 & \end{array}$$

As it is not clear how such a measurement can in fact be carried out it is not possible to calculate whether the value assigned to say $\sigma_{1x}\sigma_{2y}$ is the same in this case as it was in the previous case when σ_{1x} and σ_{2y} were measured separately on the same initial state and for the same initial hidden variables. As is clear from (14) and (15) after the measurement is carried out the final state entails spin correlations. One might refer to the measurement as a non-local measurement since starting from a local factorized state a state entailing correlations is produced.

In the weak form of the argument the values entered in the table are those that the hidden-variable theory predicts a measurement will reveal given the actual initial state, the measurement hamiltonian and the actual initial valuesof the hidden variables. Let us consider again the state (13) and choose the hidden variables to be $x_1, y_1, z_1 > 0$ and $x_2, y_2, z_2 > 0$. The values for the product operators will be assumed to be obtained by measurement of the individual operators separately. The table of values then has the form

$$\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & -1 & \end{array}$$

The operator relations in the first two rows and the first two columns are trivially satisfied. The operator relation expressed in the third column is not satisfied by these counterfactual values since the way in which the measurements are assumed to be carried out does not involve measuring these operators simultaneously. If they were to be so measured then this would conflict with the operator relations in the first two rows and columns.

The contradiction can be avoided if we assume that the value assigned to the product operators depends on which other operators are measured simultaneously. That is the product operator $\sigma_{1x}\sigma_{2y}$ can have a different value (for the same set of hidden variables) when measured with σ_{1x} and σ_{2y} than the value it has when measured with $\sigma_{1y}\sigma_{2x}$and $\sigma_{1z}\sigma_{2z}$. That is, a contextual hidden-variable theory can avoid the contradiction. This point was made by Bell [9] historically in response to the original arguments of this type and also more recently by Brown [10].

However Mermin's argument has nothing to do with non-locality if one considers the sytem under discussion to be two spin-$\frac{1}{2}$ particles in a product state. The same is not true for the argument of Peres. Peres considers a subset of six of the nine operators mentioned above and relies on the properties of the singlet spin state. (In fact Peres' argument came first and was extended by Mermin). The six are

$$\begin{array}{cc|c} \sigma_{1x} & \sigma_{2x} & -1 \\ \sigma_{2y} & \sigma_{1y} & -1 \\ \sigma_{1x}\sigma_{2y} & \sigma_{1y}\sigma_{2x} & -1 \\ \hline 1 & 1 & \end{array}$$

with products along rows and columns as shown. Again in a similar way values (or measurement outcomes) cannot be consistently assigned. Let us note that in straight quantum mechanics when $\sigma_{1x}\sigma_{2y}$ and $\sigma_{1y}\sigma_{2x}$ have definite values $\sigma_{1x}, \sigma_{2x}, \sigma_{2y}$ and σ_{1y} do not.

Since the argument of Peres concerns the singlet state the assumption 3a is not trivially satisfied. Indeed if one relaxed this assumption, as Peres states, the contradiction could be avoided. The value assigned to σ_{1x} (for example) could depend on the measurement carried out on particle two as it does explicitly in the Bohm theory. In this case the contextuality inherent in the Bohm theory is manifested as non-locality. This amounts to a denial of faithful measurement, since a unique value that is simply revealed by measurement can not then be given for the individual particle operators in Peres' example nor for the product operators in Mermin's.

6. Conclusion

The value assigned to an operator for a given set of hidden variables also depends on the quantum state in the Bohm theory. Consequently the value revealed by a measurement then depends on the hidden varibles, the initial quantum state and the measurement hamiltonian. The state-dependence of the values is the reason why both the strong and the weak forms of the arguments under consideration do not apply to the Bohm theory. The state dependence of the values also explains why even though the principle of faithful measurement does not apply in the Bohm theory it nevertheless has the same predictions for measured values. The state dependence is the origin of the contextuality displayed by the Bohm theory. In general a simultaneous eigenstate of the operators A, B and C will be different to a simultaneous eigenstate of the set A, D and E. Since the value assigned to A depends on the state, this value can be expected to be different in the two cases even if the hidden variables are the same.

All no-hidden-variable theorems proposed thus far simply exclude an unrealistic (and non-existent) set of theories. Although the no-hidden-variable arguments under consideration present their arguments in a novel, accessable and interesting way they are no exception. As the authors of these arguments undoubtedly appreciate viable hidden-variable theories may be constructed by incorporating contextuality and relaxing the requirements of faithful measurement and of locality. We simply wish to demonstrate that the Bohm theory does just this in a natural way.

Acknowledgments

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